Bin Han, Princeton University

Joint work with

Ingrid Daubechies, Amos Ron and Zuowei Shen

on

Wavelet Tight Frames From B-Splines

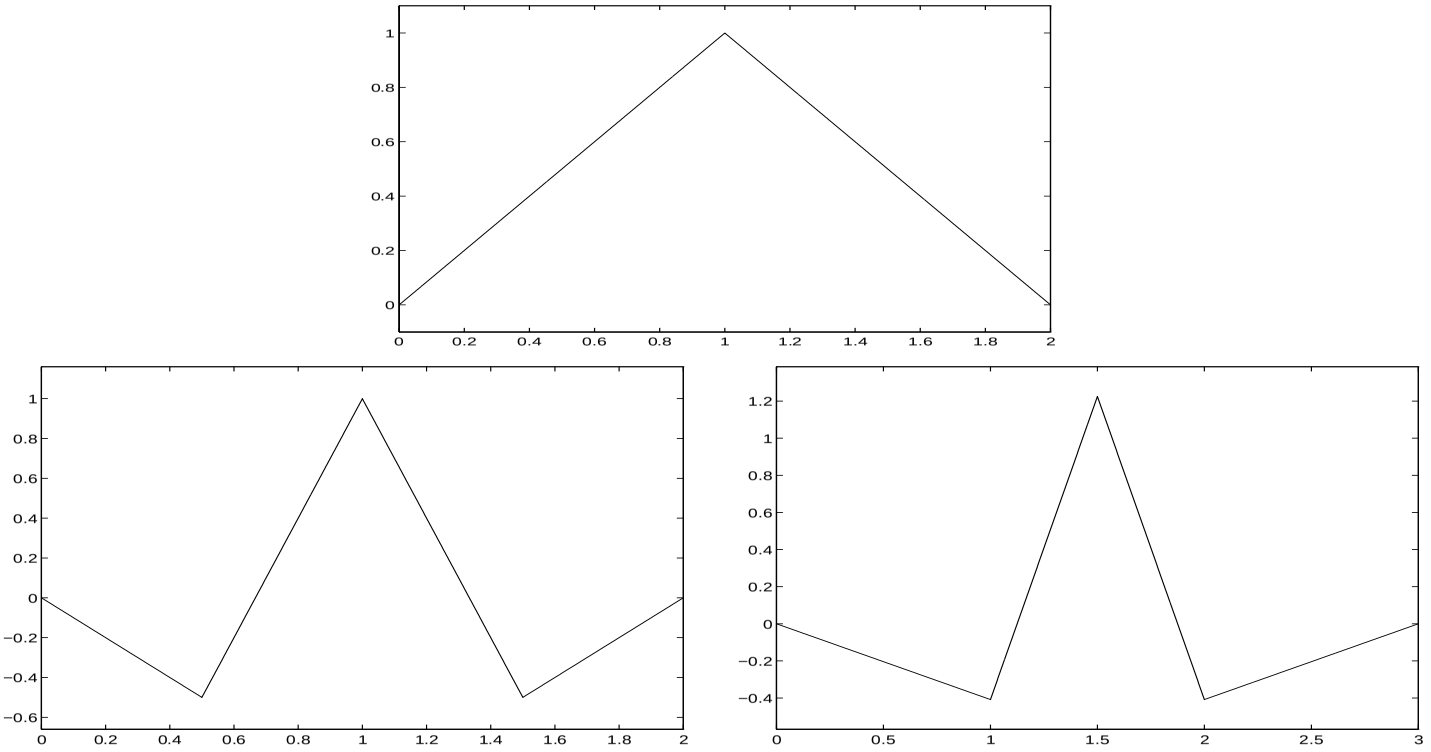
Purpose: From B-spline of order k, derive wavelet tight frames with vanishing moments of order k.

A. Abstract

Let ϕ be a refinable function with compact support. For any positive integer k, we propose a method to construct all possible multiresolution wavelet tight frames with the vanishing moments of order k from the refinable function ϕ. By applying such method to B spline functions, we shall be able to construct two wavelet functions (or three symmetric wavelet functions) from the B spline function of order k such that they generate a wavelet tight frame in $L_2(\mathbb{R})$ with vanishing moments of order k. Also wavelet tight frames from any Box spline function can be easily constructed by a project method.

B. Example 1

The B-spline function ϕ with mask $\widetilde{a}(\xi) = (1+e^{-i\xi})^2/2$:



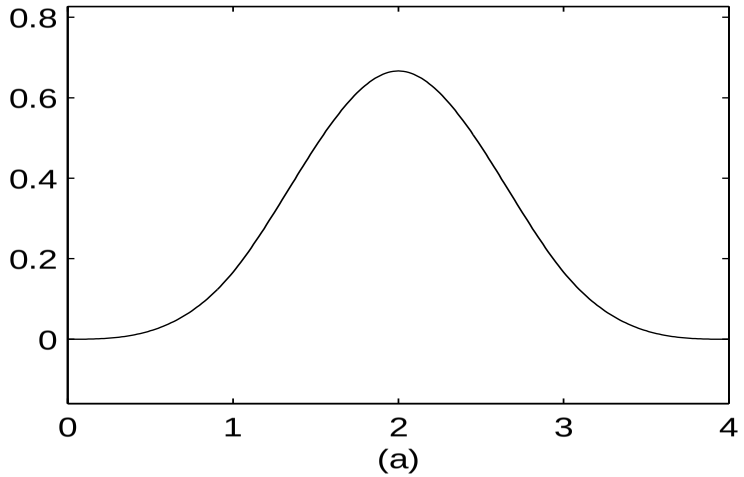
ψ^1, ψ^2 have vanishing moments 2. $\widetilde{a}^1(\xi)$ and $\widetilde{a}^2(\xi)$ are

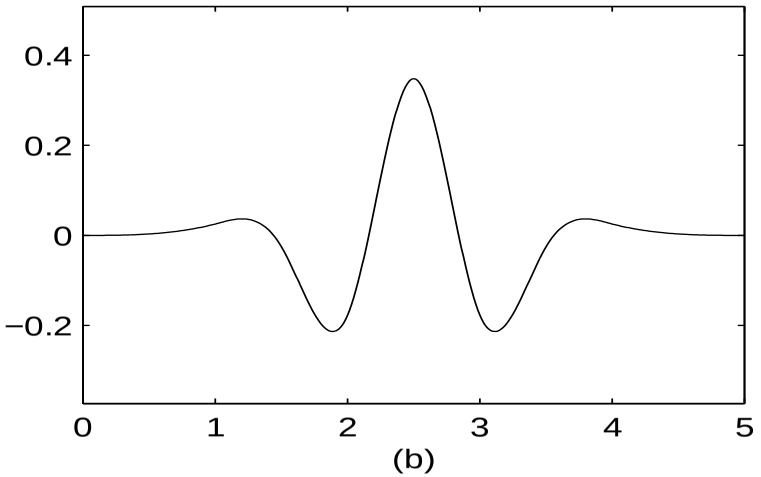
$$-\frac{1}{2}(1-e^{-i\xi})^2, \quad -\frac{\sqrt{6}}{12}(1-e^{-i\xi})^2(1+4e^{-i\xi}+e^{-i2\xi}).$$

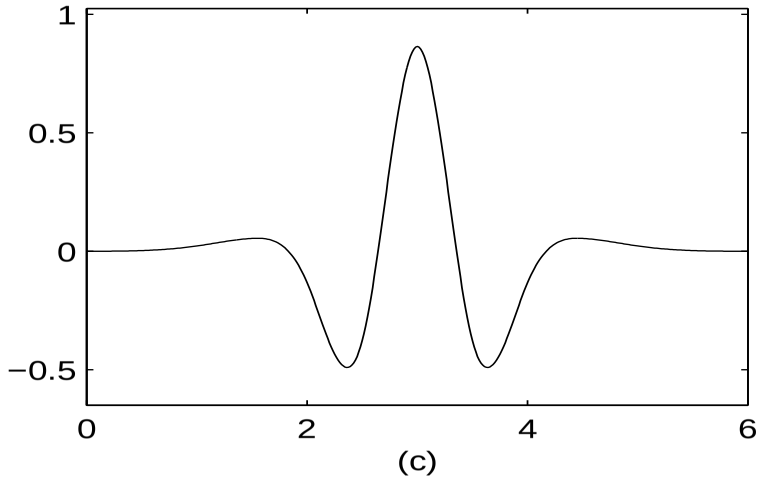
$$\|\phi\|_2^2 = 2/3, \qquad \|\psi^1\|_2^2 = 1/3, \qquad \|\psi^2\|_2^2 = 1/2.$$

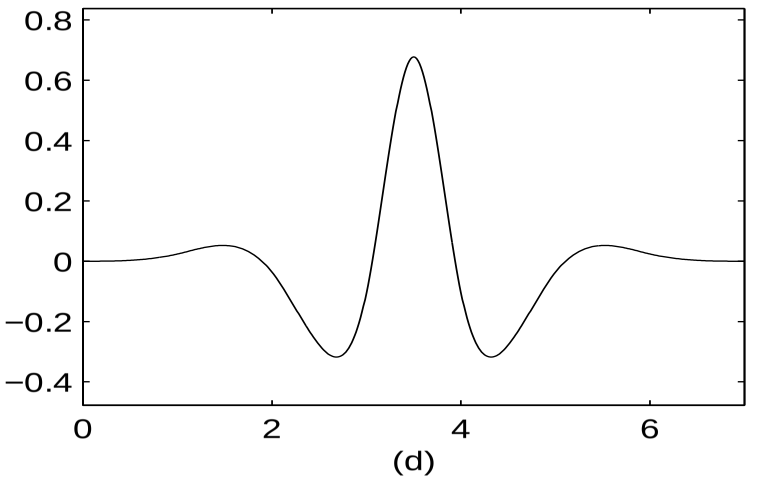
C. The B-Spline of order 4

Symmetric wavelet tight frames using 3 functions:





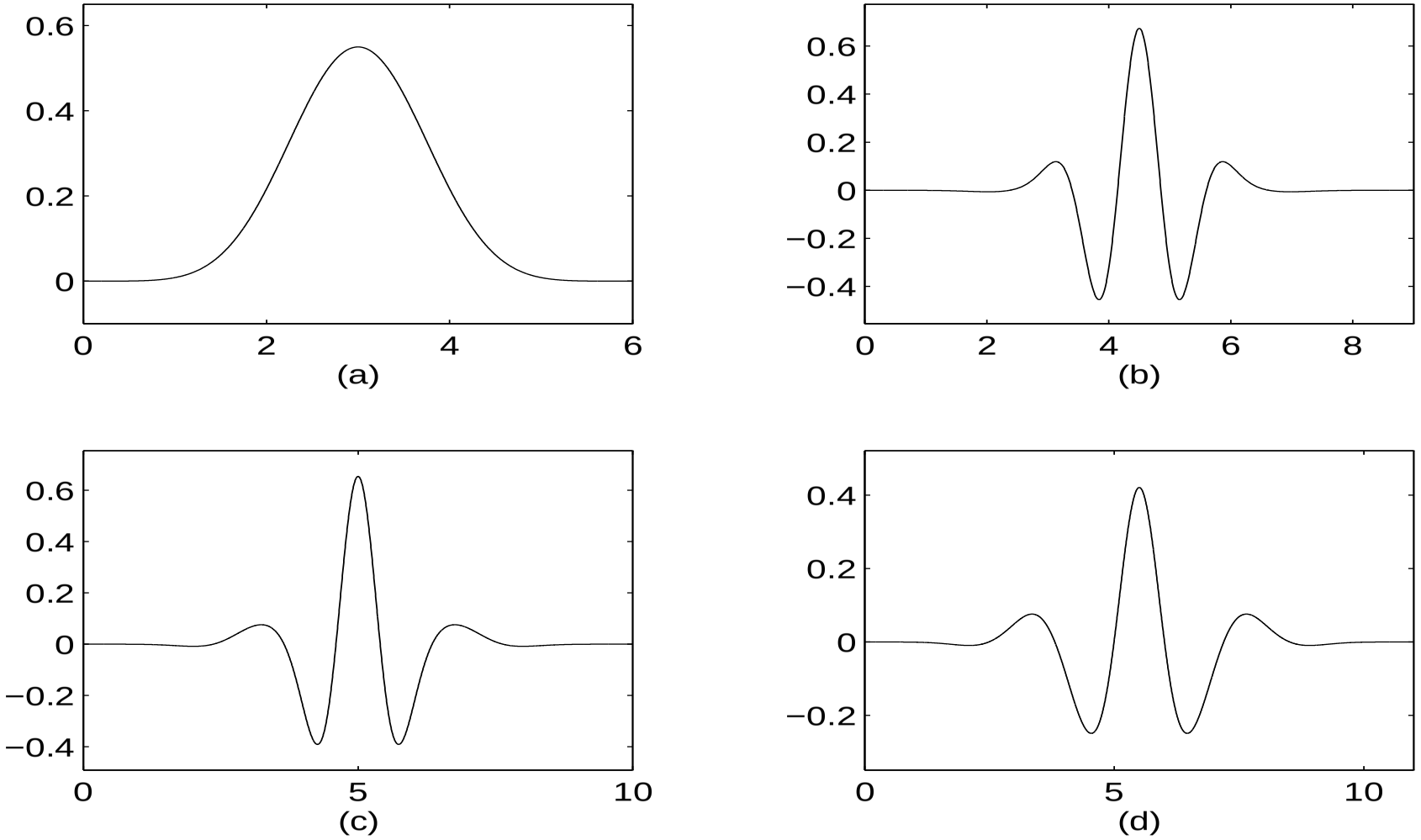




ψ^1, ψ^2, ψ^3 have vanishing moments of order 4.

D. The B-Spline of order 6

Symmetric wavelet tight frames using 3 functions:



ψ^1, ψ^2, ψ^3 have vanishing moments of order 6.